

THERMODYNAMIC AND MECHANICAL PROPERTIES OF SKELETAL MUSCLE CONTRACTION¹

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¹Paper presented at the Fifteenth Symposium on Thermophysical Properties, June 22-27, 2003, Boulder, Colorado, U.S.A.

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ABSTRACT

Such thermodynamic parameters as the change of entropy, internal energy and enthalpy were calculated in dependence on the skeletal muscle relative strain within the framework of proposed thermodynamic model. The value for the Young modulus for the skeletal muscle was estimated too. The obtained theoretical results are in a good agreement with available experimental ones for the frog skeletal muscle contraction.

KEY WORDS: skeletal muscle contraction; thermodynamic and mechanical properties

1. INTRODUCTION

Muscle, as a complex biophysical system, draws attention of many researchers. It is conditioned mainly by newly received experimental data [1-4] on muscle contraction, in particular of a force, which is developed by a fibre of skeletal muscle under loading (the so-called *equation of state*) and electrical stimulation and its dependence on length, speed, and other physical parameters. It was determined, that the obtained experimental results did not quite match the postulates of a model of skeletal muscle contraction, known as the theory of sliding [5]. Therefore it is needful to explain these experimental facts within the framework of a correct mechanism of contraction and to make adequate mathematical formulation.

First of all it should be noted, that all contraction models in one way or another are connected with the dynamics of myosin-actin complex within the sarcomere (the smallest moving unit of skeletal muscle). For example, active movable unit of a sliding model [6] is a heavy fraction of myosin (the head of a myosin molecule). Besides, the sliding model has two submodels, one of which is based on the inactivity of light fraction of myosin and activity of its heavy fraction, while the other one, on the contrary, is based on the contracting activity of light fraction with a heavy fraction functioning as a connector of myosin and actin filaments. According to another mechanism [7], the contraction of skeletal muscle is a result of twisting of myosin filaments into the tubiform structures, formed by actin filaments. The contraction is possible owing to interdomain motion of two molecule heads, which move in turns. Pollak's model [1] is based on the "stepwise" contraction of muscle fiber. Davydov's model [8] is grounded on motion as well, but that of a soliton, which stimulates the reciprocal movement of both myosin and actin filaments. There is another model [9], which is based on stimulation of α -helical segments of albumen, but unlike Davydov's model, it is focused not on the soliton motion, but on a deformation reaction of α -helix to stimulation. Though the concepts concerning the mechanism of contraction considerably differ, all the existing models are connected with the motion ("sliding," "twisting," "stepwise," "soliton" and so on). In addition, this motion should be asynchronous in different parts of a sarcomere and in different points of time to provide (during the process of contraction) constant interaction between actin and myosin. Such asynchrony means simultaneous presence in sarcomere of all possible impulse values of moving elements, regardless of what is meant under the "moving element," that is regardless of the model. Besides, these are the elements, which can be considered as one-dimensional gas of non-interacting particles (at least to a first approximation). Therefore it's obvious, that to explain experimental data on skeletal muscle contraction, one should use methods of statistical physics for one-dimensional gas of non-interacting particles.

2. MODEL AND ITS DISCUSSION

Let's consider a biological system – sarcomere (its length $\sim 2.5 \mu\text{m}$), stimulation of which results in contraction. In consequence, in such a continuum – one-dimensional gas, emerge movements with continuous or quasi-continuous set of impulses $p = \hbar k$ (\hbar is a Plank's constant and k is a wave vector), which make the sarcomere function.

As is known [10], the physical behavior of any system is determined by its statistical sum Z , taken over all probable states. In this case, these states are determined by value of a wave number k . In other words, the statistical sum is given by the formula

$$Z = \sum_k e^{-\frac{E(k)}{\Theta}}, \quad (1)$$

where the energy of a system may be written as:

$$E(k) = \frac{\hbar^2 k^2}{2m}. \quad (2)$$

Here m is a mass of active motor element; $\Theta = k_B T$, where k_B is a Boltzmann constant and T is an absolute temperature.

Substituting (2) in the formula (1), we get:

$$Z = \sum_k e^{-\frac{\hbar^2 k^2}{2m\Theta}}. \quad (3)$$

The formula above (3) is used to find the statistical sum. Since the problem is one-dimensional, in this formula we can change from summing to integration by area, where

values of k have physical significance: $\frac{4\pi}{l} \leq |k| \leq \frac{\pi}{2a}$, where l is a length of a region under review (length of sarcomere) and a is the smallest area in a system (it varies in different models). Then

$$Z = \frac{l}{\pi} \int_{\frac{4\pi}{l}}^{\frac{\pi}{2a}} e^{-\frac{\hbar^2 k^2}{2m\Theta}} dk. \quad (4)$$

Supposing

$$\frac{\hbar^2 k^2}{2m\Theta} = X^2 \quad (5)$$

we get

$$Z = 4 \cdot \frac{\Phi(X_2) - \Phi(X_1)}{X_1}, \quad (6)$$

where

$$X_1 = \frac{4\pi\hbar}{l\sqrt{2m\Theta}} = \frac{l_0}{l}, \quad (7)$$

$$X_2 = \frac{\pi\hbar}{2a\sqrt{2m\Theta}} = \frac{l_0}{8a}, \quad (8)$$

$$\Phi(X) = \int_0^X e^{-t^2} dt. \quad (9)$$

Here $l_0 = \frac{4\pi\hbar}{\sqrt{2m\Theta}}$ is a certain parameter, which has a dimension of length and can concur or be proportionate to the initial length of sarcomere. This implies that the length of sarcomere depends on temperature as in:

$$l \sim \frac{l_0}{\sqrt{T}}. \quad (10)$$

Let's estimate the value of X_2 and the function $\Phi(X_2)$, supposing $l_0 \sim 2.5 \mu\text{m}$ and $a \sim 20 \text{ nm}$ (this value corresponds to the length of the head of a myosin molecule [6] and to the length of a "skip" in Pollak's model [1]): $X_2 \sim 15.6$ and $\Phi(15.6) \sim$

$$\Phi(\infty) = \frac{\sqrt{\pi}}{2} = 0.89.$$

As regards the function $\Phi(X_1)$, we decompose into Taylor series to the second nonzero term inclusive, that is:

$$\Phi(X_1) = X_1 - \frac{X_1^3}{3}. \quad (11)$$

Then the statistical sum (3) is

$$Z = 4 \cdot \frac{0.89 - X_1 + \frac{X_1^3}{3}}{X_1},$$

or, taking into account the expression (7),

$$Z = 4 \cdot \left\{ 0.89 \frac{l}{l_0} - 1 + \frac{1}{3} \left(\frac{l_0}{l} \right)^2 \right\}. \quad (12)$$

Using the calculated statistical sum (12) we find free energy of the system:

$$F = -\Theta \cdot \ln Z = -\Theta \cdot \ln 4 - \Theta \cdot \ln \left\{ 0.89 \frac{l}{l_0} - 1 + \frac{1}{3} \left(\frac{l_0}{l} \right)^2 \right\}. \quad (13)$$

Since the change of a muscle volume on its strain in isobaric condition ($P = \text{const}$) is insignificant ($P\Delta V \approx 0$), the change of Helmholtz free energy of the system can be written as

$$\Delta F \approx -f\Delta l - S\Delta T, \quad (14)$$

where the contraction force f is concentrated in the opposite direction from the direction

of muscle contraction and is a function of its relative deformation $\varepsilon = \frac{\Delta l}{l_0}$

$$f(\varepsilon) = - \left(\frac{\partial F}{\partial l} \right)_T = f_0 \varphi(\varepsilon),$$

where

$$f_0 = \frac{k_B T}{l_0}, \quad (15)$$

$$\varphi(\varepsilon) = \frac{0.223 + 1.78\varepsilon + 3.56\varepsilon^2 + 0.89\varepsilon^3}{0.223 + 0.89\varepsilon + 2.34\varepsilon^2 + 2.56\varepsilon^3 + 0.89\varepsilon^4}. \quad (16)$$

The relation (16) is shown in Fig. 1. As we can see, it has the maximum value at $\varepsilon_c=0.25$, which, though the phases of contraction and relaxation are simultaneous, divides them. This value corresponds to the “physiological” muscle length $1.25l_0$. The increase of this length can lead to irreversible changes in the muscle, which, under such conditions, loses its ability to contract. The maximum possible lengthening of a muscle in our case is $1.9l_0$ ($\varepsilon_0=0.9$). The received theoretical curve in Fig. 1 nicely corresponds with the data of well-known classical experiments on stress-strain properties of a frog skeletal muscle, sited in the works [1, 6].

From the formulas (15) and (10) we obtain the fact, that the contraction force of a muscle depends on temperature:

$$f \sim k_B T^{3/2}. \quad (17)$$

Entropy of the system is given by the formula:

$$S(\varepsilon) = - \left(\frac{\partial F}{\partial T} \right)_l = k_B \ln \phi(\varepsilon).$$

$$\phi(\varepsilon) = \frac{0.893 + 2.68\varepsilon + 6.68\varepsilon^2 + 3.56\varepsilon^3}{(1 + \varepsilon)^2}. \quad (18)$$

The change of internal energy and enthalpy is given by the formula

$$\Delta U(\varepsilon) \approx \Delta H = - f \Delta l + T \Delta S, \quad (19)$$

where, using formulas (10) and (17), the isothermal change of the entropy of a system, can be written as:

$$\Delta S(\varepsilon) = - \int_{f_0 \varphi(\varepsilon)}^{f_0} \left(\frac{\partial l}{\partial T} \right)_f df \sim - \frac{k_B}{2} \ln \phi(\varepsilon). \quad (20)$$

The relation (20) is shown in Fig. 2. As we can see, on the strain of the muscle, the value of entropy decreases and reaches its minimum value at the point $\varepsilon_c=0.25$. However, the tendency of a strained muscle to contract is determined by a spontaneous tendency of the entropy to increase, which occurs at $\varepsilon_c < \varepsilon \leq \varepsilon_0$.

Substituting the formula (20) in the formula (19) we get:

$$\Delta U(\varepsilon) \approx \Delta H \sim -k_B T \psi(\varepsilon),$$

$$\psi(\varepsilon) = \varepsilon\varphi(\varepsilon) + \frac{\ln \varphi(\varepsilon)}{2}. \quad (21)$$

The relation (21) is shown in Fig. 3. As we can see, its minimum value is at $\varepsilon_0=0.9$, which corresponds to the maximum possible muscle strain $1.9l_0$. In the investigated area of a length change of relative muscle deformation, the change of enthalpy $\Delta H < 0$, in other words, the system, evolves heat.

Finally, muscle tension is given by the formula:

$$\sigma(\varepsilon) = \frac{f}{s} = \frac{f_0}{s} \varphi(\varepsilon). \quad (22)$$

From this we obtain the formula for its Young modulus

$$E = \left. \frac{d\sigma}{d\varepsilon} \right|_{\varepsilon=0} \approx 4 \frac{f_0}{s}, \quad (23)$$

where s is a square of a muscle crosscut. By experiments [3, 11] we know, that $\frac{f_0}{s} \approx (0.1-0.3)$ MPa (the maximum value of isometric strain of a frog skeletal muscle).

Consequently, we have $E \approx (0.4-1.2)$ MPa (for compare: $E \approx 8$ MPa for rubber).

The calculated muscle tension by the formula

$$\sigma(\varepsilon) \approx \frac{\langle E \rangle}{4} \varphi(\varepsilon) \quad (24)$$

is given in Fig. 4 (here $\langle E \rangle = 0.8$ MPa). As one can see, the obtained $\sigma(\varepsilon)$ dependence as the $f(\varepsilon)$ function (Fig. 1) has the maximum value at $\varepsilon_c = 0.25$, and further it falls with the increase of lengthening muscle.

3. RESULTS

Within the framework of the model of the skeletal muscle contraction proposed, which is based on the general principles of statistical physics, it was possible first to analytically calculate a change in the internal energy and enthalpy of this system and to estimate its Young modulus. The obtained theoretical results qualitatively well agree with the known classical experiments from the study of the mechanical properties of the frog skeletal muscle contraction. By interesting result, in our opinion, there is establishment of the temperature dependence of the sarcomere length and force of the muscle contraction, which requires further experimental examination.

REFERENCES

1. G.H. Pollak, *Muscles and Molecules: Uncovering the Principles of Biological Motion* (AIP, New York, 1990).
2. P.A. Wahr, and J.M. Metzger, *J.Physiol.* **85**: 76 (1998).
3. K.A.P. Edman, *J.Physiol.* **519**: 515 (1999).

4. M.S. Miroshnichenko, I.A. Zaloilo, D.M. Nozdrenko, and Yu.I. Prylutsky, *Physics of alive* **9**: 71 (2001).
5. V.I. Deshcherevskiy, *Mathematical models of muscle contraction* (Nauka, Moscow, 1977).
6. J. Bendoll, *Muscles, molecules and motion* (Mir, Moscow, 1970).
7. M.S. Miroshnichenko, and M.F. Shuba, *Usp. fiziol. nauk* (in Russian) **21**: 3 (1990).
8. A.S. Davydov, *Solitons in the molecular systems* (Naukova Dumka, Kiev, 1988).
9. A.D. Suprun, and Yu.B. Atmagha, *Visnyk Kyiv Univ.* (in Ukrainian) **3**: 470 (2000).
10. E.M. Lifshits, and L.P. Pitaevskiy, *Statistical physics* (Nauka, Moscow, 1978).
11. R.L. Lieber, M.E. Leonard, C.G. Brown, and C.L. Trestik, *Am.J.Physiol.* **261**: 86 (1991).

Figure Captions

Fig. 1. Calculated by formula (16) the force of the muscle contraction (referred to the amplitude value f_0) as the function of relative deformation ε .

Fig. 2. Calculated by formula (20) the change of entropy system (referred to the Boltzmann constant k_B) as the function of relative deformation ε .

Fig. 3. Calculated by formula (21) the change of internal energy system (referred to the thermal energy $k_B T$) as the function of relative deformation ε .

Fig. 4. Calculated by formula (24) the muscle tension as the function of relative deformation ε .







