

A Fundamental Boiling Point Equation (BPE) of Pure Fluids.
Applicability to Wagner VPE of Water and Comparison with
Kretzschmar-Oguchi VPE.

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ABSTRACT

This work has brought out the existence of a fundamental BPE based on any empirical VPE in logarithmic form. The proposed BPE as a backward equation of the VPE can be used in any thermodynamic model avoiding the usual numerical iteration technique. The inconsistent error between VPE and BPE is less than 0.4 per cent. When the permissible error is extremely small, then three adjustable parameters are adequate in most cases to satisfy this requirement. The proposed pair of equations needs significantly less computing time than any accurate iteration algorithm.

This paper also addresses the problem of achieving accurate interrelation between vapor pressure and boiling point experimental data of pure fluids and an effective and significant universality relating the two parameters, in dimensionless form is obtained. Convertible pairs of VPE and BPE can be derived from this implicit equation that can also be used to verify any set of experimental vapor pressure data.

The proposed model has been applied to water here and is shown to do an excellent job representing data accurately. Wagner international VPE of the steam with a 10 coefficient BPE as a backward equation can work accurately with inconsistent error less than the extremely small permissible value of 0.003 per cent. A comparison of the proposed pair with the Kretzschmar - Oguchi implicit bi-quadratic equation reveals the pros and cons of both models.

KEYWORDS : Vapor Pressure Equation, Boiling Point Equation, Wagner international VPE of the steam, Kretzschmar–Oguchi VPE.

1. INTRODUCTION

Due to the great significance of the accurate description of the vapor–liquid saturation boundary in many engineering applications, numerous Vapor Pressure Equations (VPE) have appeared in the literature.

Antoine type equations [1] are the most common but not the best and Wagner type equations [2] are the most accurate. Ambrose [3], proposed a VPE using Chebyshev polynomials. Gomez-Thodos [4], Frost and Kalkwarf, [5], Lee-Kesler, [6], perform satisfactorily although generally fail to describe the critical region adequately. Goodwin, [7], included non-analytic behavior at the critical point. Iglesias-Silva et al, [8], have been proposed a VPE representing data from the triple to the critical point, using a correlation technique from heat transfer and fluid mechanics. Low temperature VPE by King and Al-Najjar, [9], have been obtained integrating the Clausius-Clapeyron equation. Extended studies by Scott and Osborn, [10, 11], have shown that the Cox equation, [12], can adequately represent experimental data from the triple point to 3 bar.

In the present work a reliable method is provided that permits any explicit in vapor pressure form VPE to become explicit in boiling point, too. The Boiling Point Equation (BPE) is valid over the entire range from the triple point to the critical point provided the auxiliary VPE is valid too. The practical implications of this study can be significant taking into account the existence of numerous VPE and the lack of a sufficient BPE to work as a pair with them avoiding numerical iteration or similar techniques.

Experimental and smoothed data for the ten substances coming from the sources listed in Table 1 of APPENDIX have been used to establish the proposed method. The critical temperature and pressure T_c and P_c respectively, the reduced temperature of the triple point, $Tr_t = T_t/T_c$ and the natural logarithm of the reduced pressure of the triple point, $\ln(P_{r_t})$, $P_{r_t} = P_t/P_c$ for these materials are tabulated on the same Table.

2. DEVELOPMENT AND FEATURES OF THE FUNDAMENTAL BPE.

Considering the problem of finding dimensionless and uniform scale for all substances for the entire temperature and pressure range, from the triple to the critical point, we introduce the following temperature θ and pressure π functional forms :

$$\theta = \frac{T - T_t}{T_c - T_t} = \frac{Tr - Tr_t}{1 - Tr_t} \quad (1)$$

$$\pi = \frac{\ln\left(\frac{P}{P_t}\right)}{\ln\left(\frac{P_c}{P_t}\right)} = 1 - \frac{\ln(P_r)}{\ln(P_{r_t})} \quad (2)$$

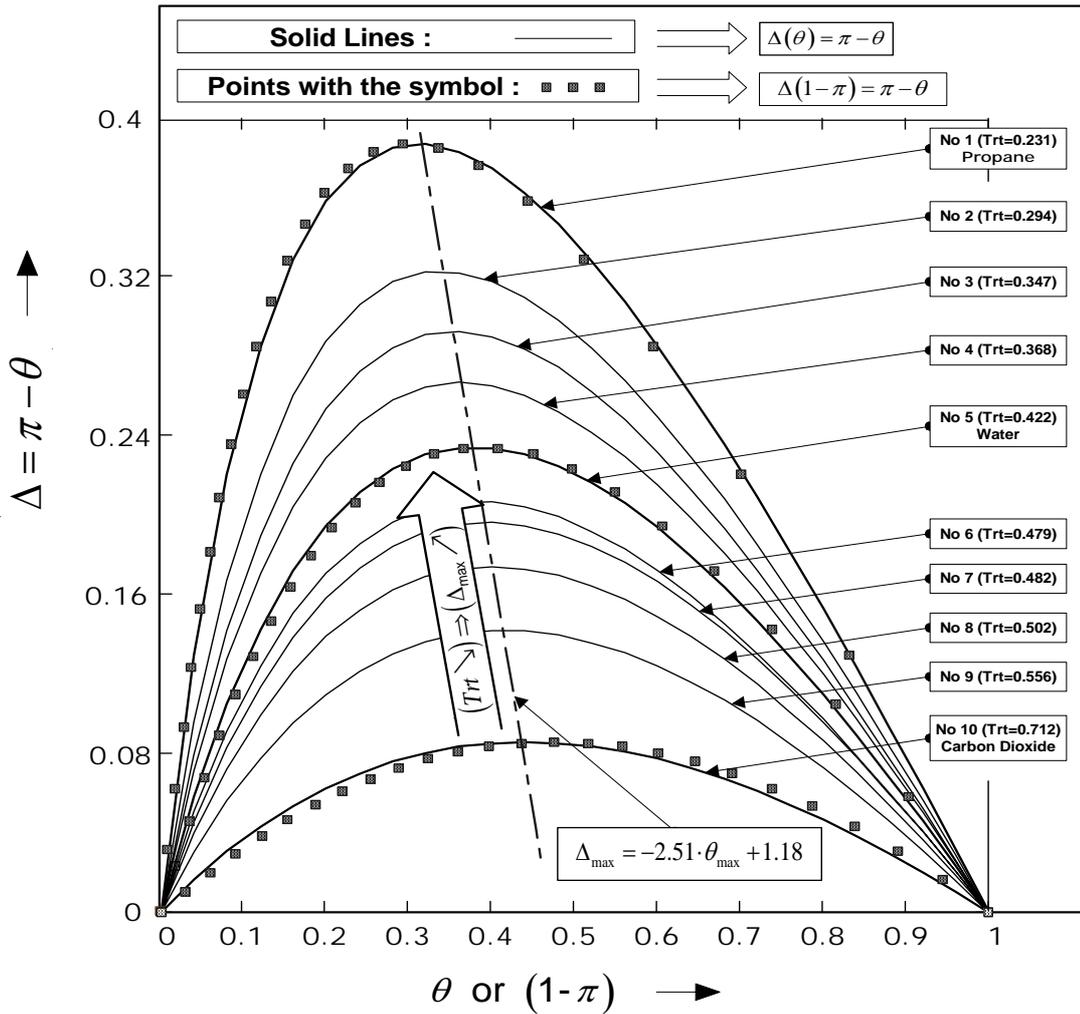


Figure 1. The difference $\Delta = \pi - \theta$ in terms of θ for the ten fluids of Table 1 (solid lines) and in terms of $1 - \pi$ for Propane (No1), Water (No 5) and Carbon Dioxide (No3), (box symbol).

Defining the difference $\pi - \theta$ as Δ :

$$\Delta(\theta) = \pi - \theta \quad (3)$$

and using the vapor pressure data for the substances of Table 1 the graphical representation of the function $\Delta(\theta)$ is shown on Figure 1.

Some important remarks and results:

The greater the triple point reduced temperature Tr_i is, the closer the $\Delta(\theta)$ curve is to the x-axis. Carbon dioxide ($Tr_i = 0.712$) and Propane ($Tr_i = 0.231$) represent the lower and upper limits in this family of curves respectively.

All the curves have the same shape and a general function relating $\Delta(\theta) = \pi - \theta$ and θ should exist. A rough approximation of this function $\Delta(\theta)$ for any substance is the relationship :

$$\Delta(\theta) = \frac{\theta \cdot (1 - \theta)}{A_1 \cdot (1 - \theta) + A_2 \cdot \theta} \quad (4)$$

The auxiliary functions A_1 and A_2 have been correlated in terms of the reduced triple point temperature Trt as follows :

$$A_1(Trt) = a_1 + a_2 \cdot Trt + a_3 \cdot Trt^2 + a_4 \cdot Trt^3 \quad (4a)$$

$$\frac{A_2(Trt)}{A_1(Trt)} = a_5 + \frac{a_6}{Trt} \quad (4b)$$

Table 2 of APPENDIX gives the values of the coefficients $a_i, i = 1 \dots 6$. The accuracy of these correlations for A_1 and A_2 are shown on Figs 2 and 3 respectively.

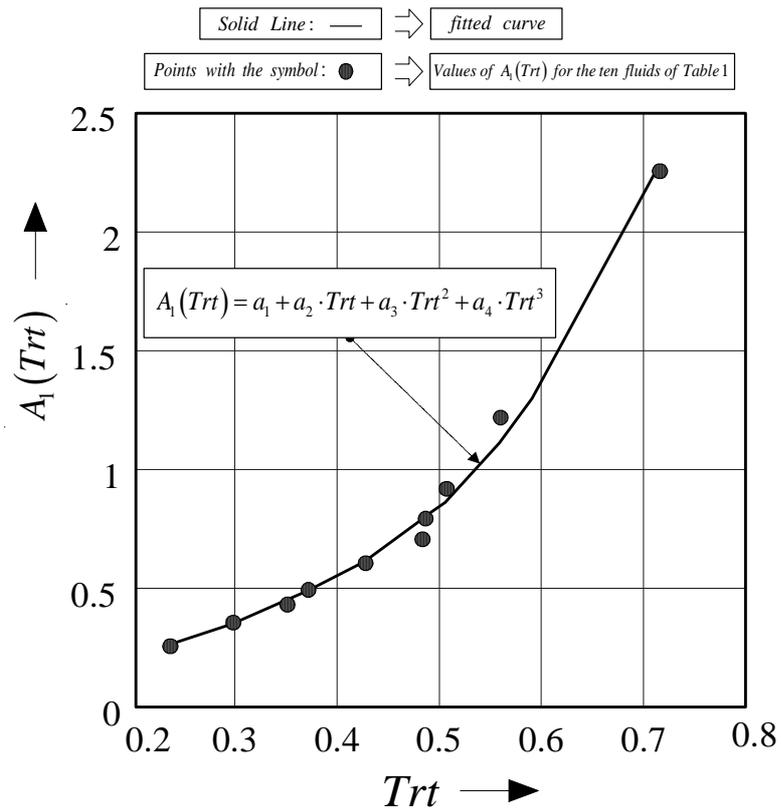


Figure 2. The auxiliary function $A_1(Trt)$ of Eq 4 in terms of Trt . The coefficients $a_i, i = 1 \dots 4$ of this function are tabulated in Table 2.

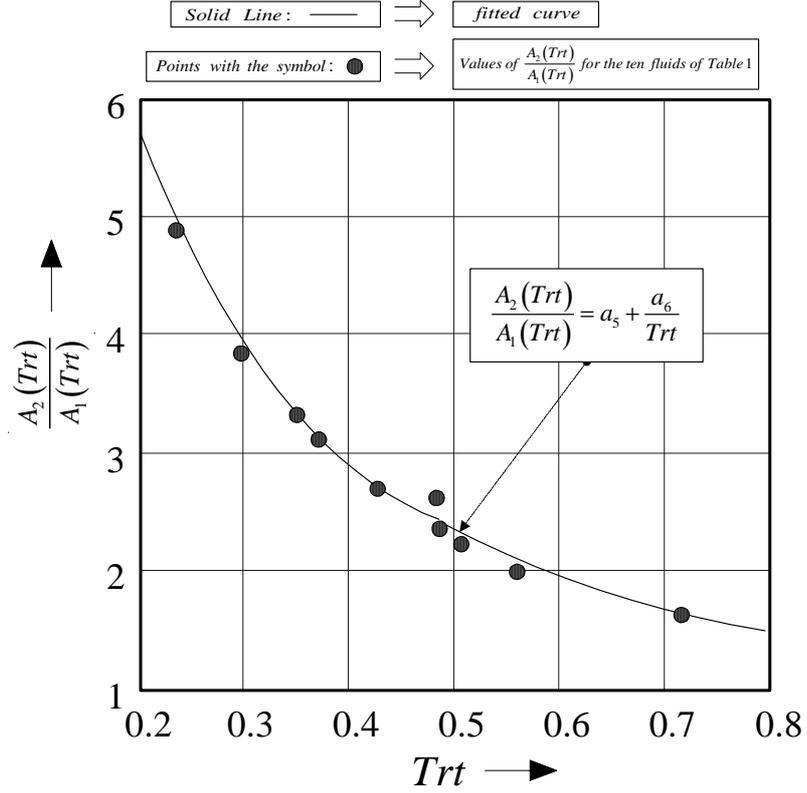


Figure 3. The ratio $\frac{A_2(Trt)}{A_1(Trt)}$ of auxiliary functions of Eq 4 in terms of Trt .
The coefficients $a_i, i = 5 \dots 6$ of this function are tabulated in Table 2.

The local maximum of all $\Delta(\theta)$ curves of Fig. 1 obey a general rule approximated by the line (dash dot line on Fig 1) :

$$(\pi - \theta)_{\max} = a_7 \cdot \theta_{\max} + a_8 \quad (4c)$$

Table 2 of APPENDIX gives the values of the coefficients $a_i, i = 7 \dots 8$. The last column of Table 1 gives the values of θ_{\max} for the considered fluids.

Plotting of $\Delta = \pi - \theta$ as a function of $1 - \pi$:

$$\Delta(1 - \pi) = \pi - \theta \quad (5)$$

it is shown that the two curves $\Delta(\theta)$ and $\Delta(1 - \pi)$ “almost” coincide for all substances : To demonstrate this property three substances have been selected, i.e. Propane (No1), Water (No2) and Carbon Dioxide (No10) and the corresponding $\Delta(1 - \pi)$ curves are represented on Fig 1 by points marked with the box symbol. This remarkable property can be expressed by the equation :

$$\Delta(\theta) = \Delta(1-\pi) \quad (6)$$

and shows that a thermodynamic interrelation connects the dimensionless temperature θ and pressure π .

In this case the local maximum of the two curves $\Delta(\theta)$ and $\Delta(1-\pi)$ coincide the relation $\theta = 1-\pi$ is valid and the coefficients of equation (4c) will be : $a_7 = -2$, $a_8 = 1$. In addition, the auxiliary functions A_1 and A_2 of equation (4)

can be obtained in a simpler way : $A_1 = \frac{\theta_{\max}^2}{1-2\cdot\theta_{\max}}$ and $A_2 = A_1 + 1$.

Convertible pairs of VPE and BPE can be derived from the implicit equation (6), creating functional forms of $\Delta(\theta)$ and $\Delta(1-\pi)$ according to the required degree of accuracy and inconsistency error.

Combination of equations (2) and (3) gives :

$$\Delta(\theta) = 1 - \theta - \frac{\ln \text{Pr}}{\ln \text{Pr}_t} \quad (7a)$$

If $\ln \text{Pr}$ is expressed in terms of Tr by any empirical VPE in logarithmic form, named $\ln \text{Pr}(Tr)$, then equation (1) allows substitution of Tr by $(1-\theta)\cdot Tr_t + Tr$, in equation (7a) as follows:

$$\Delta(\theta) = 1 - \theta - \frac{\ln \text{Pr}[(1-\theta)\cdot Tr_t + Tr_t]}{\ln \text{Pr}_t} \quad (7b)$$

Equation (6) permits the following transformation of equation (7b) in terms of $1-\pi$:

$$\Delta(1-\pi) = \pi - \frac{\ln \text{Pr}[\pi\cdot Tr_t + Tr_t]}{\ln \text{Pr}_t} \quad (8)$$

Comparison of equations (5) and (8) gives :

$$\pi - \theta = \pi - \frac{\ln \text{Pr}(\pi\cdot Tr_t + Tr_t)}{\ln \text{Pr}_t} \quad (9)$$

or

$$\theta = \frac{\ln \text{Pr}(\pi\cdot Tr_t + Tr_t)}{\ln \text{Pr}_t} \quad (10)$$

Equation (10) can be expressed in terms of Tr and Pr instead of θ and π as follows :

$$Tr = Tr_i + cn \cdot \ln Pr(cn \cdot \ln(Pr) + Tr_i) \quad (11)$$

where

$$cn = \frac{1 - Tr_i}{\ln Pr_i} \quad (11a)$$

Equation (11) is necessarily satisfied at the critical ($\pi = 1, \theta = 1$) and triple point ($\pi = 0, \theta = 0$). Its ability to correlate the v.p. data over the entire temperature range (from the triple to the critical point) depends on the quality of the selected VPE and can be any VPE proposed in the literature expressed in logarithmic form, $\ln Pr(Tr)$.

3. VERIFICATION OF METHOD USING IAPWS SKELETON TABLES OF WATER AND CONSISTENCY TASK OF IAPWS-IF97.

Wagner type VPE has become popular for numerous applications involving various classes of chemical compounds to provide a high quality fit for vapor pressure data over the entire temperature range. [15]

Of particular significance is the Wagner type international VPE of the steam, Saul and Wagner [16], converted to ITS-90 temperatures by Wagner and Pruss [17] and in this work it is used with the proposed BPE, equation (11), as a convertible equation pair to demonstrate their pros and cons.

For the water the parameters of Eq 11 are :

$$Tr_i = 0.422, \quad \ln Pr_i = -10.493 \quad \text{and} \quad cn = \frac{1 - Tr_i}{\ln Pr_i} = -0.0550702495$$

The quality of Eq. (11) is illustrated in the diagram of Fig. 4. It shows the deviation of the calculated boiling point values from the corresponding temperature values of the International Skeleton Tables IST-85 in its version of 1994 [19]/ (IST-85, Rev. 1994) :

$$err_T = \left(1 - \frac{T_s(P)}{T} \right) \cdot 100\% \quad (12)$$

The maximum deviation does not exceed ± 0.4 per cent.

The accuracy of the predicted BP values with eq. (11) can be upgraded introducing a correction factor q as follows :

$$Tr = Tr_i + cn \cdot q \cdot \ln Pr(cn \cdot q \cdot \ln(Pr) + Tr_i) \quad (13)$$

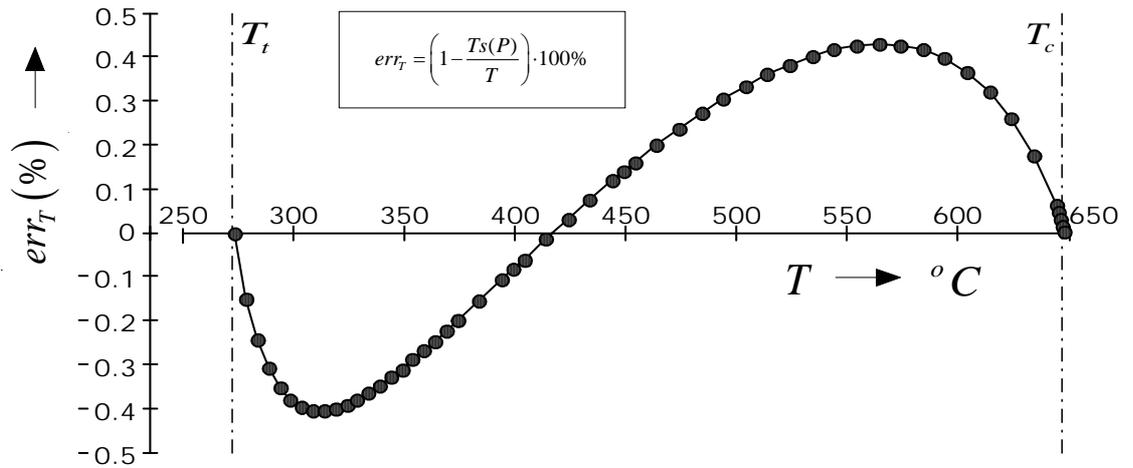


Figure 4. The deviation err_T of the calculated boiling point values for water from the corresponding values of the International Skeleton Tables (IST-85, Rev. 1994).

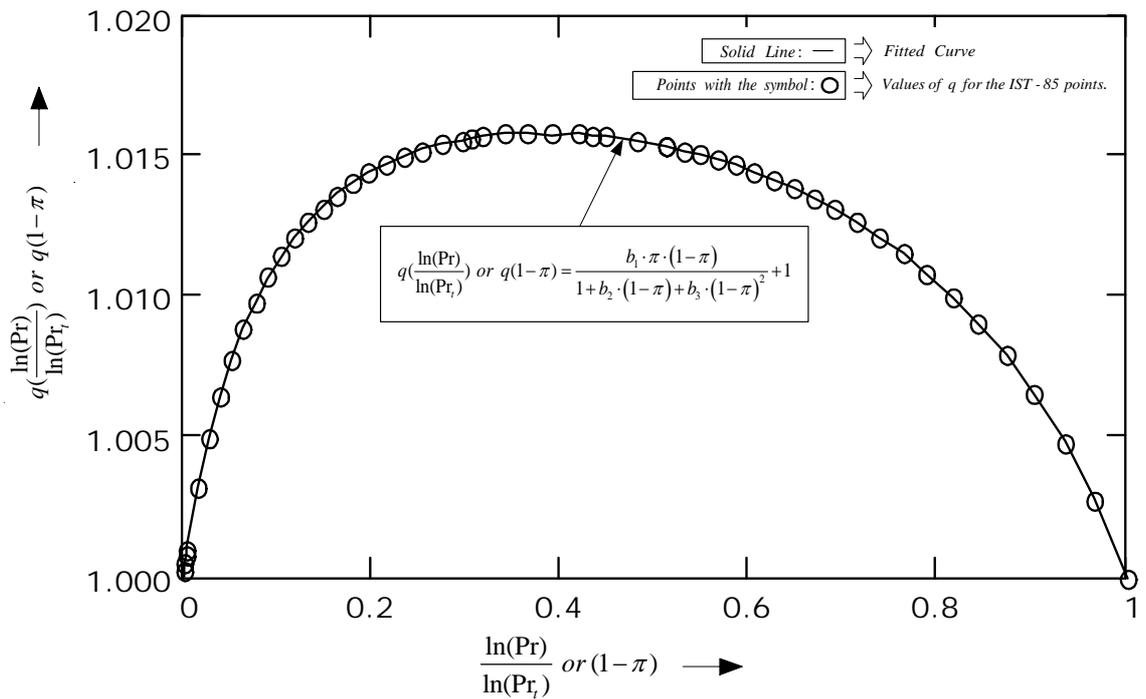


Figure 5. Comparison of the correction factor q used in Eq 13 calculated with Eq 14 (solid line), $Q\left(\frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)}\right)$, and the corresponding values coming from IST-85 points (circle symbol).

Calculating the numerical values of q for the 55 experimental data points of the Skeleton Tables IST-85 [19] the results are shown on Fig. 5 with the circle symbol.

An equation with three adjustable coefficients for q in terms of $1 - \pi = \frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)}$, read :

$$Q\left(\frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)}\right) \text{ or } Q(1 - \pi) = \frac{b_1 \cdot \pi \cdot (1 - \pi)}{1 + b_2 \cdot (1 - \pi) + b_3 \cdot (1 - \pi)^2} + 1 \quad (14)$$

and allows the calculation of the BP with sufficient accuracy. Figure 5 shows the comparison between the experimental data and the corresponding values calculated from Eq. 14 in terms of the ratio $\frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)}$.

Fig 6 shows the deviation err_T of the calculated boiling point values with equations (13) and (14) from the corresponding values of the Skeleton Tables IST-85 [19].

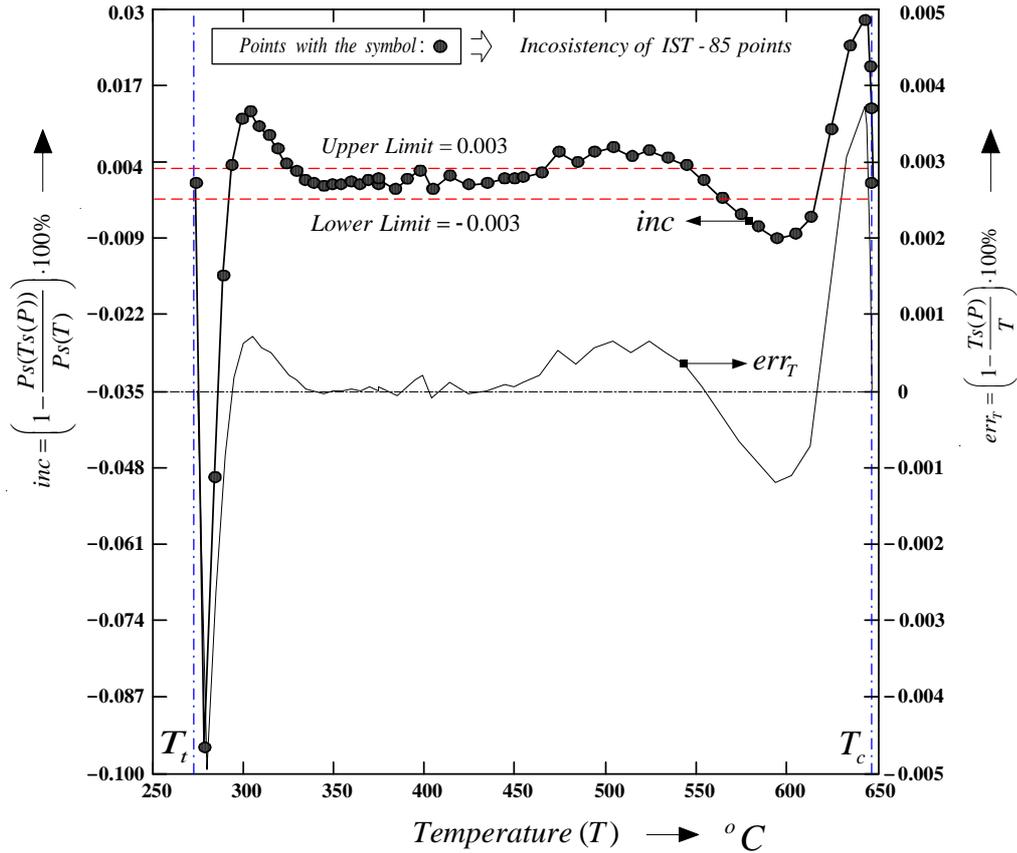


Figure 6. The deviation err_T of the BPE for water (Eq 13 with the auxiliary Eq 14) and the inconsistent error, inc , when it is used with the Wagner VPE as a convertible pair.

Consistency at vapor–liquid saturation boundary.

The permissible numerical inconsistencies between the basic VPE and backward BPE is defined in Section 5.1 (item 2(c)) of [20] as :“The saturation pressure calculated from the saturation-temperature equation $T_s(P)$ was not allowed to deviate by more than $\Delta P_s = \pm 0.003$ percent from the P_s value determined from the saturation-pressure equation $P_s(T)$.”. On Fig. 7 are shown the VPE and BPE curves, the experimental saturation line (Skeleton Tables data), as well as the calculated quantities of the experimental point (T, P) : $P_s(T)$, $P_s(T_s(P))$, $T_s(P)$ and $T_s(P_s(T))$.

If $\Delta P_s = P_s(T) - P_s(T_s(P))$, then the inconsistency error is defined as :

$$inc = \left(1 - \frac{P_s(T_s(P))}{P_s(T)} \right) \cdot 100\% \quad (15)$$

In addition of the BP deviation err_T , Figure 6 also shows the numerical inconsistencies when BPE/Eq (13) with the auxiliary Eq (14) and Wagner VPE are used as a convertible pair. It can be seen that the inconsistent error, inc , can be less than the extremely small permissible value of 0.003 per cent except for the points near the critical and triple point.

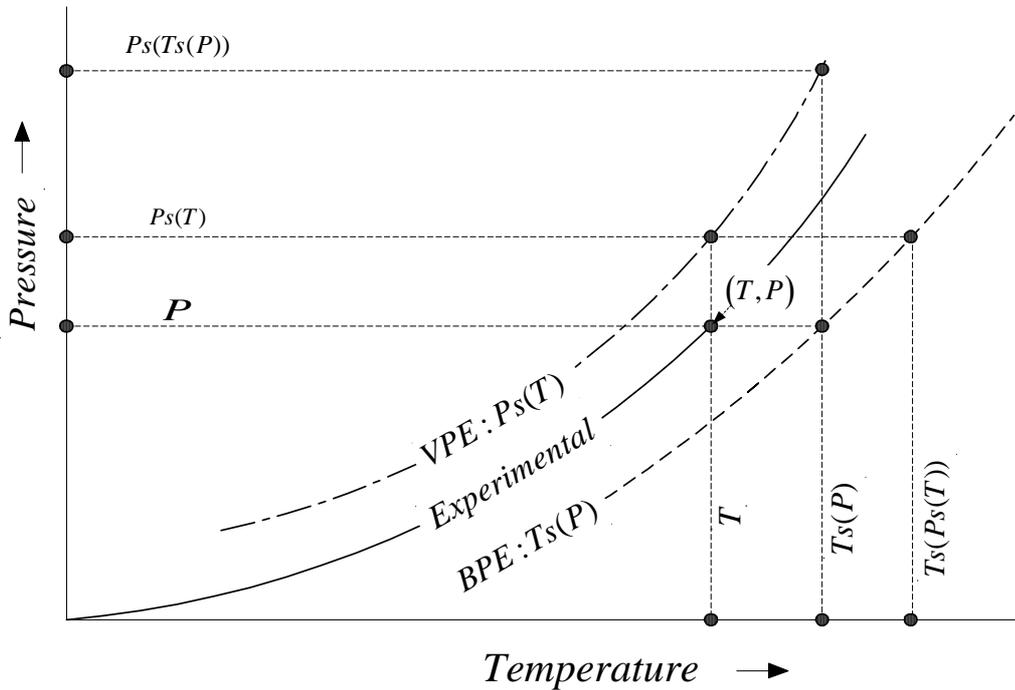


Figure 7. The VPE and BPE curves, the experimental saturation curve (IST-85, Rev. 1994) and the calculated quantities of the experimental point (T, P) : $P_s(T)$, $P_s(T_s(P))$, $T_s(P)$ and $T_s(P_s(T))$.

4. DEVELOPMENT OF AN ACCURATE VPE-BPE AND COMPLIANCY WITH CONSISTENCY TASK (WAGNER, VPE/ Eq. 18, BPE).

Based on a survey made by the “IAPWS/ Subcommittee on Industrial Calculations ” among the international power-cycle companies and related industries, the BPE has been included in the set of the sixteen more important property functions of steam and it has among them the highest average frequency of use (30.7%).

The main task of this part of the work is to derive a BPE coming from equation (5), which as a pair with the Wagner VPE fulfills the extremely small permissible value 0.003 of consistency criterion. Equation (5) can be expressed as :

$$\theta = \pi - \Delta(1 - \pi) \quad (16)$$

The function $\Delta(1 - \pi)$ can be expressed in terms of $1 - \pi$ with high accuracy using the following 10 coefficients form :

$$\Delta\left(\frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)}\right) \text{ or } \Delta(1 - \pi) = \frac{\pi \cdot (1 - \pi) \cdot \sum_{n=1}^5 c_n \cdot (1 - \pi)^{n-1}}{1 + \sum_{n=6}^{10} c_n \cdot (1 - \pi)^{n-5}} \quad (17)$$

Table 3 of APPENDIX gives the values of the coefficients c_i , $i = 1..10$. Combination of equations (16) and (17) and transformation from the parameters θ and π to the parameters Tr and Pr gives :

$$\frac{Tr - Tr_t}{1 - Tr_t} = 1 - \frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)} - \frac{\left(1 - \frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)}\right) \cdot \left(\frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)}\right) \cdot \sum_{n=1}^5 c_n \cdot \left(\frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)}\right)^{n-1}}{1 + \sum_{n=6}^{10} c_n \cdot \left(\frac{\ln(\text{Pr})}{\ln(\text{Pr}_t)}\right)^{n-5}} \quad (18)$$

Eq 18/ BPE, as a pair with Wagner VPE fulfill the extremely small permissible value 0.003 of consistency criterion. The results are presented in the next part in comparison with the Kretzschmar and Oguchi equations.

5. COMPARISON OF THE PROPOSED PAIR (WAGNER, VPE/ Eq.18, BPE) WITH KRETZSCHMAR-OGUCHI PAIR.

Recently Kretzschmar and Oguchi (K-O) have presented an implicit bi-quadratic equation for water, which provides both explicit VPE and BPE and fulfils the requirements of IST-85, Rev. 1994, [20], [21].

To clarify how the deviation and the consistency test are applied in the next paragraphs, Fig. 8 shows the common K-O curve of VPE and BPE, the experimental

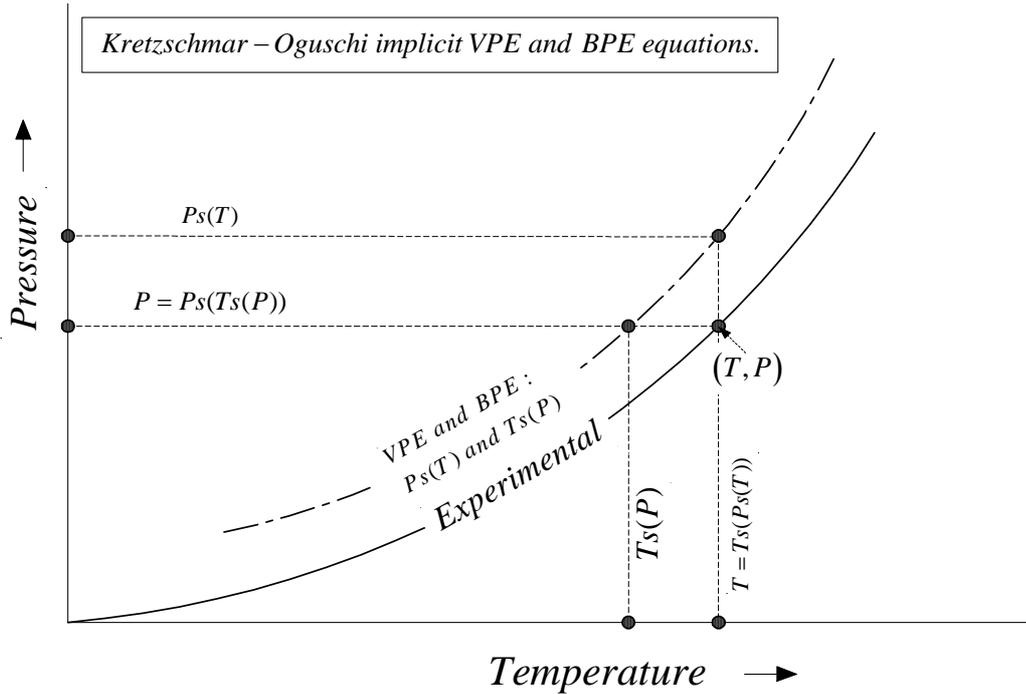


Figure 8. The Kretzschmar and Oguchi (K-O) implicit VPE/BPE equation for water, the experimental saturation curve (IST-85, Rev. 1994) and the calculated quantities of the experimental point (T, P) : $P_s(T)$, $P_s(T_s(P))$, $T_s(P)$ and $T_s(P_s(T))$.

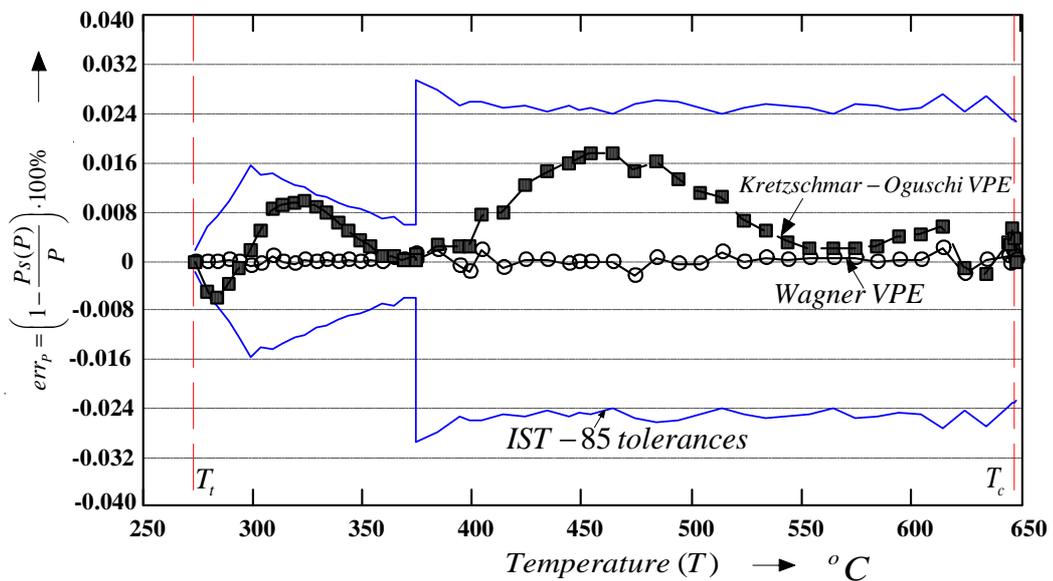


Figure 9. The deviation err_p of the calculated vapor pressure values according Wagner VPE from the corresponding values of the Skeleton Tables IST-85 (solid line with circle symbol) in comparison with the deviation of the K-O BPE (dash line with box symbol) with the associated tolerances.

saturation line (Skeleton Tables data), as well as the calculated quantities of the experimental point $(T, P) : P_s(T), P_s(T_s(P)) \equiv P, T_s(P)$ and $T_s(P_s(T)) \equiv T$.

Fig 9 shows the deviation err_p of the calculated vapor pressure values according Wagner VPE from the corresponding values of the Skeleton Tables IST-85 (solid line with circle symbol) in comparison with the deviation of the K-O BPE (dash line with solid box symbol) with the associated tolerances.

Fig 10 shows the deviation err_T of the calculated boiling point values according the proposed BPE (Eq 18) from the corresponding values of the Skeleton Tables IST-85 (solid line with circle symbol) in comparison with the deviation of the K-O BPE (dash line with solid box symbol).

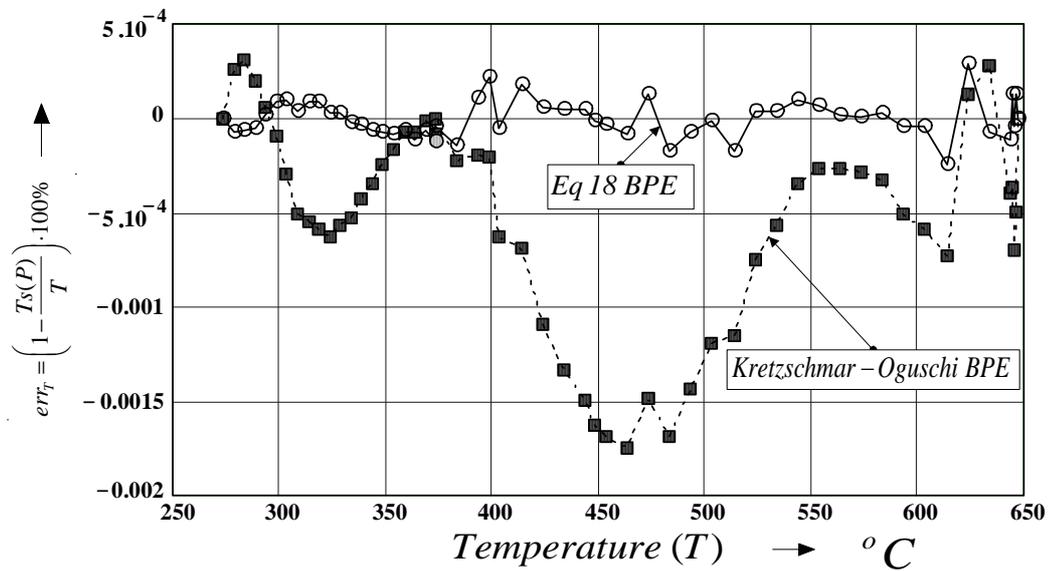


Figure 10. The deviation err_T of the calculated boiling point values according the proposed BPE (Eq 18) from the corresponding values of the Skeleton Tables IST-85 (solid line with circle symbol) in comparison with the deviation of the K-O BPE (dash line with box symbol).

Consistency test with the Eq 15.

The consistency test for the K-O pair following Eq 15 criterion is shown on Fig 11 in comparison with the Wagner/VPE- Eq 18/BPE pair. All the points for the Wagner/VPE- Eq 18/BPE comply to the consistency criterion (solid line with solid circle symbol). Most of the points for the K-O pair does not comply to the consistency criterion (dash line with solid box symbol) due to :

$$|inc| = \left| \left(1 - \frac{P_s(T_s(P))}{P_s(T)} \right) \right| \cdot 100 = \left| \left(1 - \frac{P}{P_s(T)} \right) \right| \cdot 100 > 0.003 \quad (19)$$

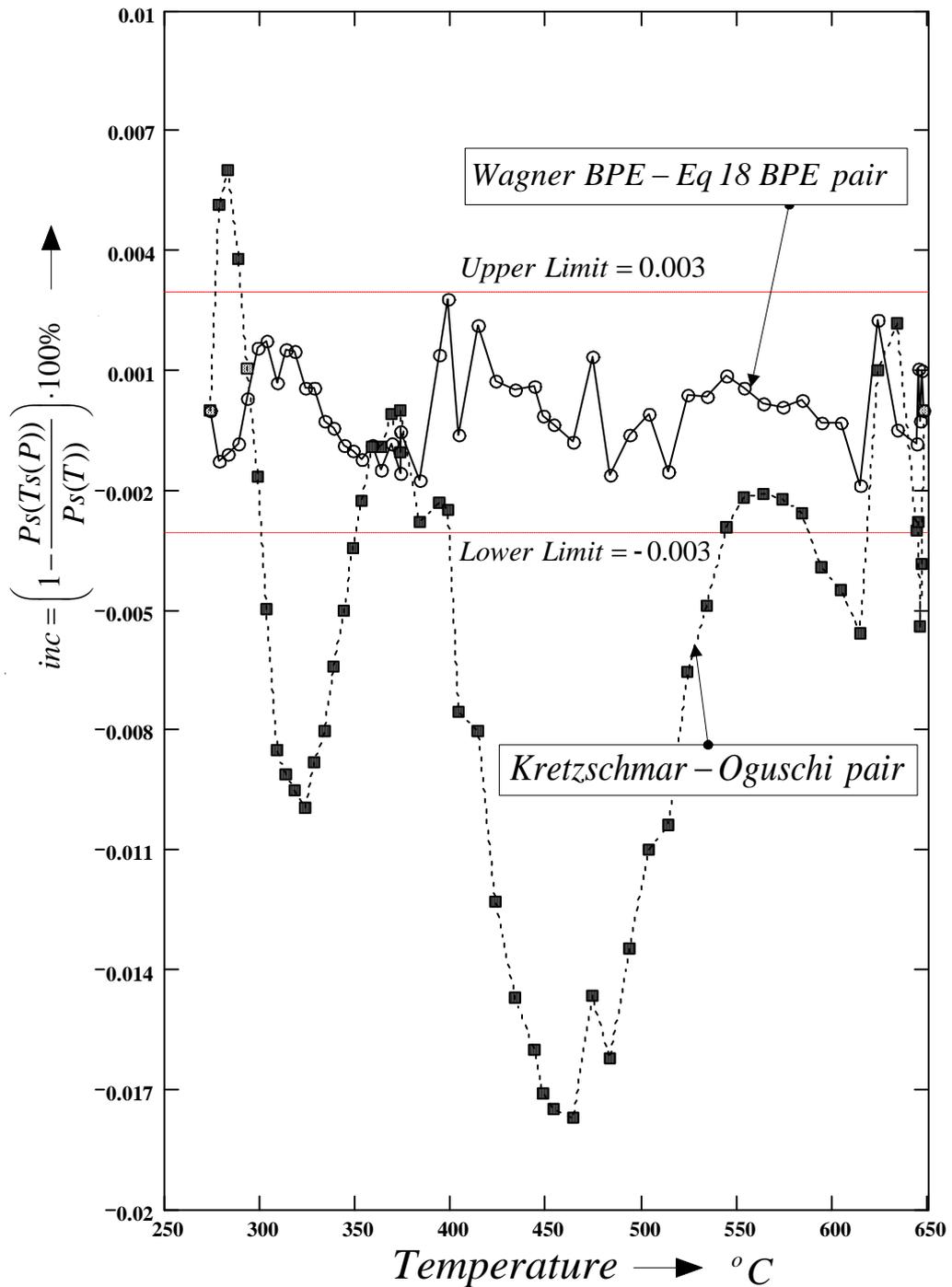


Figure 11. The inconsistent error, inc , according Eq 15, when BPE (Eq 18) it is used with the Wagner VPE as a convertible pair (solid line with circle symbol) in comparison with the deviation of the K-O BPE (dash line with box symbol).

Certainly since the VPE and BPE of the K-O pair have been derived from the same implicit equation, both equations are completely consistent with each other and when “jumping” back and forth from a single experimental point no numerical problem is caused. However in most cases, for example, when calculating the turbine-expansion line of a power-cycle process, the “jumping” back and forth between the basic and the

backward equations happens to different points and in the opinion of the author the consistency should be considered regarding the actual temperature and pressure values of the experimental point according to Eq 15. Then K-O pair transfers from one point that the temperature is known (taken as the real value) to the next point as consistency error the quantity $\left| \left(1 - \frac{P}{P_s(T)} \right) \right| \cdot 100$ and from one point that the pressure is known to the next point as consistency error the quantity $\left| \left(1 - \frac{T}{T_s(P)} \right) \right| \cdot 100$.

Restoration of Consistency test with the following criterion :

$$inc^* = \left(1 - \frac{P_s(T_s(P))}{P} \right) \cdot 100\% \quad (20)$$

In addition to Eq 15, to redress the contradiction of the previous paragraph, the two pairs of equations are recompared using the consistency criterion of Eq 20. In this case the K-O pair of equations are completely consistent with each other i.e. for any considered point, $inc^* (\%) = 0$.

Applying the same criterion to the Wagner/VPE- Eq 18/BPE pair gives as shown on Fig 12 the expected unavoidable inconsistencies of any set of independent equations (solid line with circle symbol). However, it can be seen from Fig 12 that in this case the numerical consistency is extremely small, less than the half of the permitted inconsistency.

6. CONCLUSIONS

Introducing appropriate dimensionless forms for the vapor pressure (VP) and the boiling point temperature (BP), a method has been developed that allows to derive from the experimental data analysis of pure fluids an effective and significant universality relating the two dimensionless parameters, expressed by Eq 6.

Convertible pairs of VPE and BPE can be derived from the implicit Eq 6, creating functional forms of $\Delta(\theta)$ and $\Delta(1-\pi)$ according to the required degree of accuracy and inconsistency error of any application. In addition, Eq 6 may be used for a first verification of any set of experimental vapor pressure data.

The same method has brought out the existence of a fundamental, single explicit BPE, expressed in dimensionless form by Eq 11. Some advantages of the proposed BPE are:

- Any explicit VPE with this BPE as a backward equation can be used as a pair in any thermodynamic model avoiding the usual numerical iteration technique.
- The BPE is expressed in terms of the same numerical coefficients of the VPE and the use of any complex approximation algorithm is not required.

- The proposed pair of equations needs significantly less computing time than any accurate iteration algorithm.
- The inconsistent error between VPE and BPE is less than 0.4 per cent. When the permissible error is extremely small, then three adjustable parameters are adequate in most cases to satisfy this requirement.

The correlative capabilities of the proposed method have been demonstrated starting from the function $\Delta(1-\pi)$ and fitting a 10 coefficient BPE on the Skeleton Tables data. The inconsistent error of the Wagner VPE of water with this BPE (Eq 18) as a backward equation is less than the extremely small permissible value of 0.003 per cent [20]. So the validity and the usefulness of the proposed method are confirmed.

A comparison of the proposed pair with the Kretschmar - Oguchi implicit bi-quadratic equation reveals the pros and cons of both models.

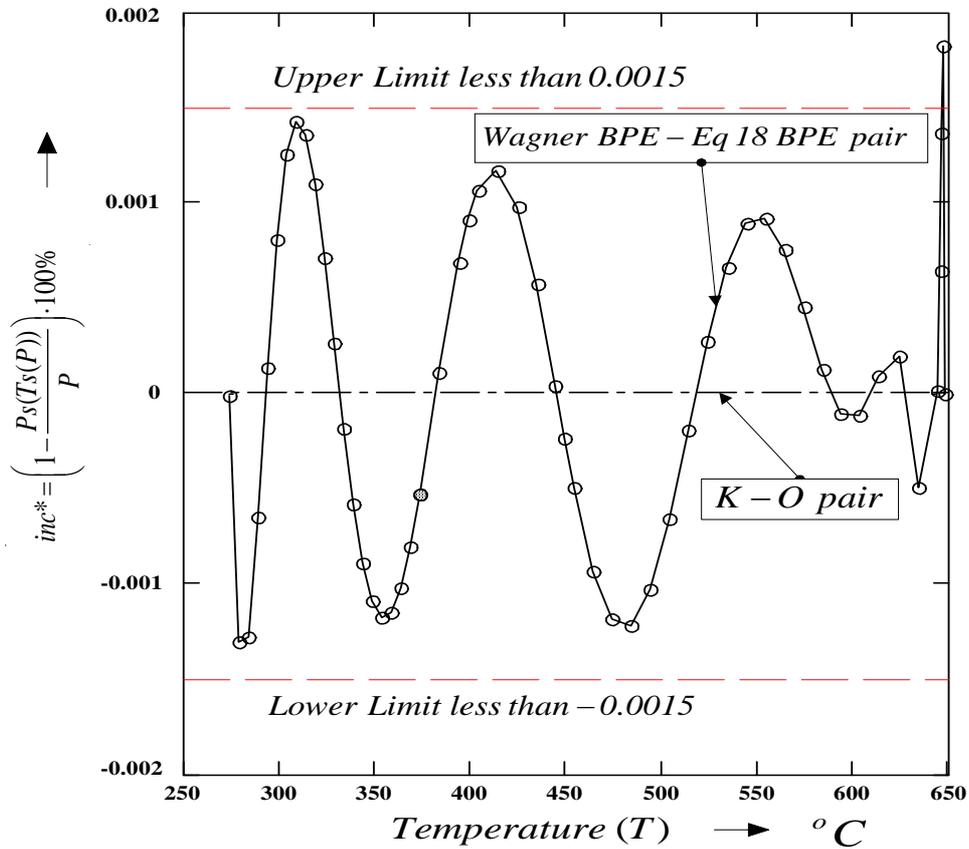


Figure 12. The inconsistent error, inc^* , according Eq 20, when BPE (Eq 18) it is used with the Wagner VPE as a convertible pair (solid line with circle symbol) in comparison with the deviation of the K-O BPE (dash line with box symbol).

NOMENCLATURE

a_i	The coefficients of the Eq 4a, 4b and 4c.
b_i	The coefficients of the Eq 14
c_i	The coefficients of the Eq 18
cn	A constant of Eq 11
err_p	Percentage deviation of the pressure
err_T	Percentage deviation of the temperature
inc	The inconsistent error according to Eq 15
inc^*	The inconsistent error according to Eq 20
$\ln Pr(Tr)$	A VPE in logarithmic form
q	Correction factor of Eq 13
$A_1(Trt)$	Auxiliary function of Eq 4
$A_2(Trt)$	Auxiliary function of Eq 4
BPE	Boiling Point Equation
P	Pressure
Pr	Reduced Pressure
$Ps(T)$	VPE
$Q\left(\frac{\ln(Pr)}{\ln(Pr_i)}\right)$	A correlation of q in terms of $\frac{\ln(Pr)}{\ln(Pr_i)}$
T	Temperature
Tr	Reduced Temperature
$Ts(P)$	BPE
VPE	Vapor Pressure Equation
Greek symbol	
θ	Dimensionless Temperature
π	Dimensionless Pressure
Δ	The difference $\Delta = \pi - \theta$
Subscripts	
c	Critical Point
max	Maximum
t	Triple Point
i	Counter

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APPENDIX (TABLES)

Table 1 The listing of the examined substances and their constants.

No	Substance	Source	Tc (K)	Pc (bar)	Trt	ln(Prt)	θ_{\max}
1	Propane	Younglove et al, [13]	369.85	42.471	0.231	-23.936	0.311
2	Ethane	Younglove et al, [13]	305.34	48.714	0.294	-15.4	0.337
3	1-chloro-1,1-difluoroethane	Reid et al, [14]	409.6	43.3	0.347	-14.055	0.354
4	Ethylene	McGarry, [15]	282.4	50.4	0.368	-10.634	0.361
5	Water	Wagner et al, [16, 17]	647.096	220.64	0.422	-10.493	0.378
6	Ethylenediamine	Reid et al, [14]	593	62.8	0.479	-9.143	0.382
7	Ammonia	Haar et al, [18]	405.5	113.5	0.482	-7.542	0.394
8	Nitrogen	McGarry, [15]	126.2	33.9	0.502	-5.577	0.400
9	Argon	McGarry, [15]	150.8	48.7	0.556	-4.265	0.415
10	Carbon dioxide	McGarry, [15]	304.1	73.8	0.712	-2.653	0.439

Table 2 Coefficients of Eq. (4a), (4b) and (4c).

a_1	-0.34481583
a_2	4.3206974
a_3	-10.712836
a_4	13.744027
a_5	0.095817646
a_6	1.1167371
a_7	-2.51
a_8	1.18

Table 3 Coefficients of Eq. (14).

b_1	0.23315699914345
b_2	9.510471696356
b_3	-7.882296722769

Table 4 Coefficients of Eq. (17) and (18).

c_1	1.327744530267	c_6	93.329909721071
c_2	129.29806638384	c_7	674.413470068365
c_3	934.892639601876	c_8	857.008327088004
c_4	-256.603262181868	c_9	2485.79072376054
c_5	4672.4201303621	c_{10}	4819.8594777844