

Calculation of Effective Thermal Conductivity of Highly Porous Two-phase Materials

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This paper deals with an empirical correlation for easy estimation of effective thermal conductivity (ETC) of highly porous two-phase systems based on experimental data available. Recently, Calmidi & Mahajan [1] and Boomsma & Poulidakos [2] have presented geometrical models for highly porous metal foams. However, our approach is based on numerical simulations in which ETC of a random two-phase medium is being comprised of contributions from both the phases. To take account of highly conducting phase, non-linear flow of heat flux lines, random distribution of the phases and non-uniform shape of particles, a correlation term F is introduced. F is computed using the proposed relation and experimental data available in the literature. An effort is being made to correlate the values of F estimated numerically, in terms of ratios of thermal conductivity of the phases, geometry of the material used and the porosity of the matrix. Best fitted relation for F with ratios of thermal conductivity and porosity is presented here. ETC of highly porous systems has been calculated using it in the proposed expression. A very good agreement is observed with experimental data of metallic foams and the results of independent calculation. Comparison of the proposed relation with different models [2,3] has also been made.

Key words: Effective thermal conductivity; metallic foams; numerical simulation; correlation factor.

1. INTRODUCTION

Search for the materials of industrial importance and their characterisation is always needed and is a challenging task for engineers and physicists. High fluid porosity metal foams have long been used in design of aircraft wing structure in the aerospace industry, catalytic surfaces for chemical reactions, core structure for high strength panels, and containment matrix and burn rate exchanger for solid propellants. The ETC depends on various characteristics of the material such as thermal conductivity, porosity, size of the particles and packing of the constituent phases. Accounting for all these factors in order to predict ETC is a complex affair. In the literature one finds several efforts [4-9] in which the situation has been simplified by assuming that the particles are of specific shape and arranged in a particular geometry within the continuous phase.

Recently, Calmidi & Mahajan [1] and Boomsma & Poulikakos [2] developed models utilizing geometrical estimate for calculation of the effective thermal conductivity specifically for metallic foams saturated with a fluid. For high porosity metal foams Calmidi & Mahajan [1] presented a one dimensional heat conduction model considering the porous medium to be formed of a two dimensional array of hexagonal cells. Whereas Boomsma & Poulikakos [2] proposed a three dimensional model using metal foam structure in the form of tetrakaidecahedral cells with cubic nodes at the intersection of two nodes. Both the models involved a geometric parameter that was evaluated using the experimental data. Recently, Bhattacharya et al [3] has extended the analysis of Calmidi & Mahajan [1] with a circular intersection, which results in a six fold rotational symmetry.

In the present paper we have tried to develop empirical relation for quick estimation of effective thermal conductivity of highly porous systems. In order to incorporate varying individual geometries and non-linear flow of heat flux lines generated by the difference in thermal conductivities of constituent phases, a correlation term F has been introduced. Expression for correlation term F has been obtained by simulating experimental data reported in the literature. The present approach is simple and provides wider applicability of the proposed relation and enhances its ability to predict correctly the ETC of real two-phase systems.

2. MATHEMATICAL FORMULATION

In literature [10-13] one finds that the effective thermal conductivity of a composite is an additive property. Considering various components as resistors one can take a combination of such resistors to predict effective thermal conductivity. This is a common practice adopted to predict effective thermal conductivity from the thermal conductivity of the constituent phases. Accepting the similarity, a relation is proposed by us in the following manner.

Consider a two-phase medium made up of solid material (subscript s) and a fluid (subscript f) filling the pore space having volume fractions $(1-\phi)$ and ϕ respectively. The matrix is supposed to be made up of layers oriented parallel and perpendicular to the direction of heat flow alternately. The effective thermal conductivity of parallel layers ($\lambda_{||}$) is given by the weighted arithmetic mean and that

of perpendicular layers (λ_{\perp}) by weighted harmonic mean. The corresponding expressions are

$$\lambda_{\parallel} = \phi \lambda_f + (1 - \phi) \lambda_s \quad (1)$$

and

$$\lambda_{\perp} = \lambda_f \lambda_s / [(1 - \phi) \lambda_f + \phi \lambda_s] \quad (2)$$

These are upper and lower bounds on the effective thermal conductivity (λ_e) of a two-phase system, therefore, λ_e will obviously lie between these two limits. As these relations do not predict the effective thermal conductivity of real two-phase systems correctly, a different kind of weighted geometric mean is proposed as

$$\lambda_e = \lambda_{\parallel}^F \lambda_{\perp}^{(1-F)} \quad (3)$$

Where F is assumed to be numerical correlation factor and may be defined as Fth fraction of the material oriented in the direction of heat flow and remaining (1-F)th fraction is oriented perpendicular to the direction of heat flow. ETC of a two-phase system is found to depend on the ratio (λ_s/λ_f) of thermal conductivity of the constituent phases, where λ_s is the thermal conductivity of solid phase and λ_f is the thermal conductivity of fluid phase. A higher ratio favours a larger fraction of the thermal conductivity in a direction perpendicular to heat flow.

We have solved equation (3) mathematically for F on substituting λ_{\parallel} and λ_{\perp} from equations (1) & (2) as

$$F = \frac{\ln [\phi \cdot \lambda_e/\lambda_f + (1 - \phi) \lambda_e/\lambda_s]}{\ln [1 + \phi (1 - \phi) \{ \lambda_f/\lambda_s + \lambda_s/\lambda_f - 2 \}]} \quad (4)$$

The correlation factor F may be a function of ratio of thermal conductivity of constituent phases and porosity of the system. Therefore, the variation of F against $R = \phi^{1/2} \ln (\lambda_s/\lambda_f)$ is plotted in fig 1. It is observed that F increases roughly linearly with increasing R. We have used a curve fitting technique and found that the expression

$$F = 0.3227 + 0.0615 \phi^{1/2} \ln (\lambda_s/\lambda_f) \quad (5)$$

$$\text{having } R'^2 = 0.9176.$$

Best fitted the curve in fig. 1. It is also observed from experimental results that the expression (5) does not represent the true state of affairs of a real system.

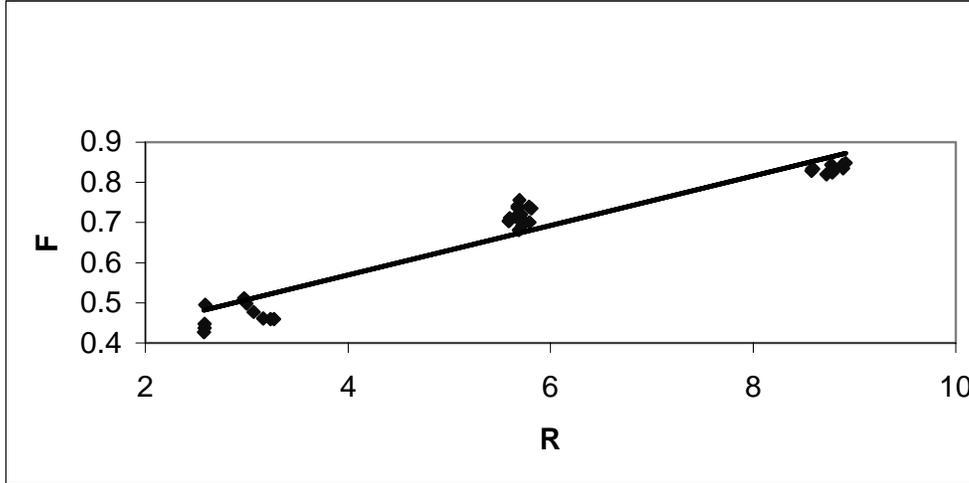


Figure 1. Variation of F with R

In addition to the physical parameters such as porosity and the ratio of thermal conductivity of the constituent phases, F is also found to depend upon various other parameters. Such parameters are given below:

2.1. Shape factor. If l and L are respective dimensions (areal) of resistors (slabs), perpendicular and parallel to the direction of heat flow then shape factor is given by l/L. For spherical particles this value is unity. For granular systems where the particles are oriented in all possible directions, its average value is also unity [14].

2.2. Sphericity of particles. Angularity of grains in a two-phase system greatly alters its thermo-physical properties. Experiments show that the behaviour of a system packed with non-spherical particles is radically different to those involving spherical particles. Sphericity is a measure of the roundness of the particles. Wedell [15] has defined it as s/S , where s is the surface area of a sphere having the same volume as that of the particle, S being the actual surface area. For spherical particles its value is unity and for particles of any other shape sphericity < 1 . An account of particle-to-particle conduction of heat in a system will require knowledge of sphericity. As sphericity decreases particle-to-particle conduction decreases.

2.3. Formation resistivity factor. The flow of energy depends on the geometrical characteristics of the medium also. These characteristics have been found [14,16&17] to be associated with a dimensional factor called formation resistivity factor.

Thus for practical utilisation, we have to modify equation (5) by incorporating some correction term C. Accommodating all the parameters, the most appropriate expression for F was found to be

$$F = C \{0.3227 + 0.0615 \phi^{1/2} \ln (\lambda_s/\lambda_f)\} \quad (6)$$

Correction term C is calculated using equation (6) and data available in table (1) for various systems and is plotted in figures 2-6 for several relationships of λ_s/λ_f and ϕ . It is also interesting to see from figure 2 that the value of C increases to a certain extent

as the ratio of thermal conductivity of the constituent phases increases after that C decreases as the ratio increases. Here average values of C are 0.8511, 1.1056, 1.1676 and 0.8854 for aluminium-air, aluminium-water, reticulated vitreous carbon (RVC)-air and reticulated vitreous carbon (RVC)-water systems respectively.

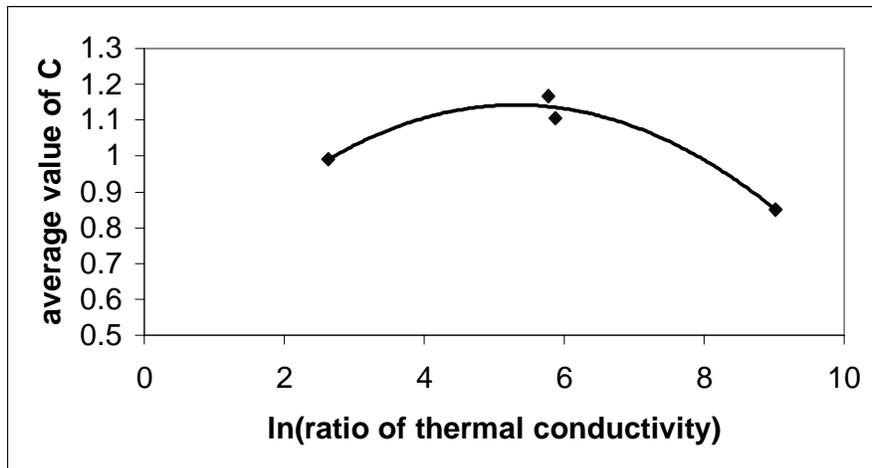


Figure 2. Variation of C with $\ln(\lambda_s/\lambda_f)$

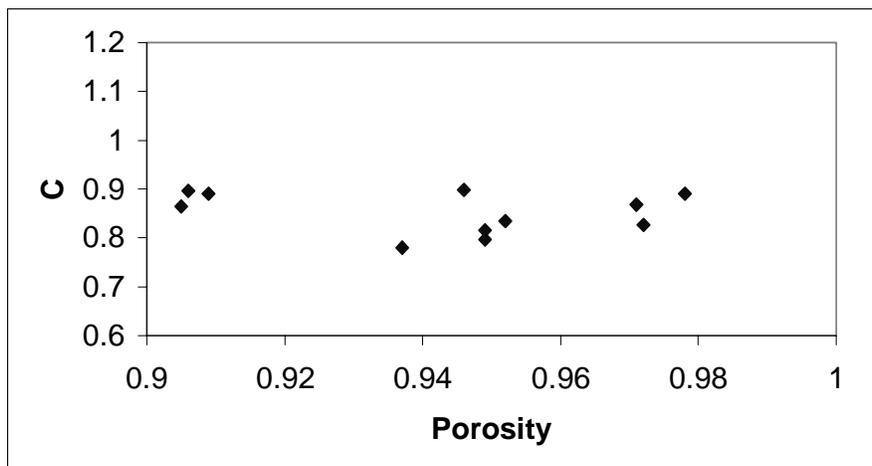


Figure 3. Variation of C with ϕ in aluminium-air system

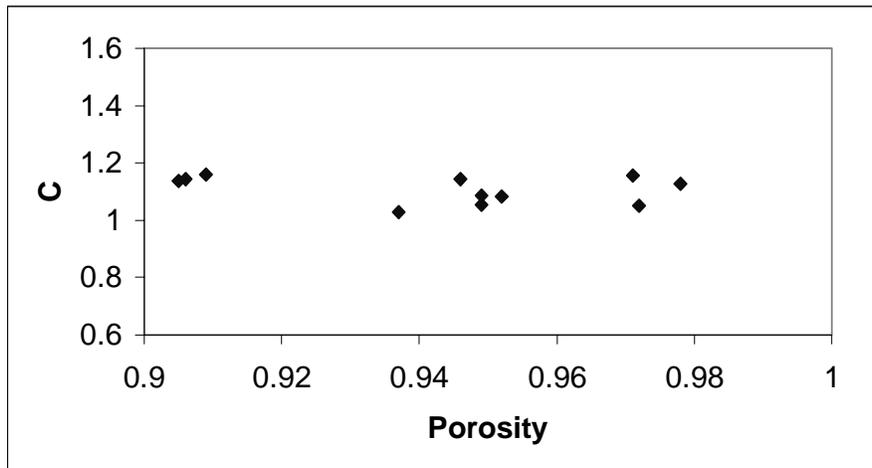


Figure 4. Variation of C with ϕ in aluminium-water system

Figures 3 to 6 show the variation of C with porosity for two separate cases (aluminium-fluid and RVC-fluid systems). It is observed on examining the figures that variation in C with porosity is nearly constant in metallic foams; whereas in non-metallic foams C increases as porosity increases. Now, using equation (6) in equation (3), effective thermal conductivity of any type of the systems can be computed.

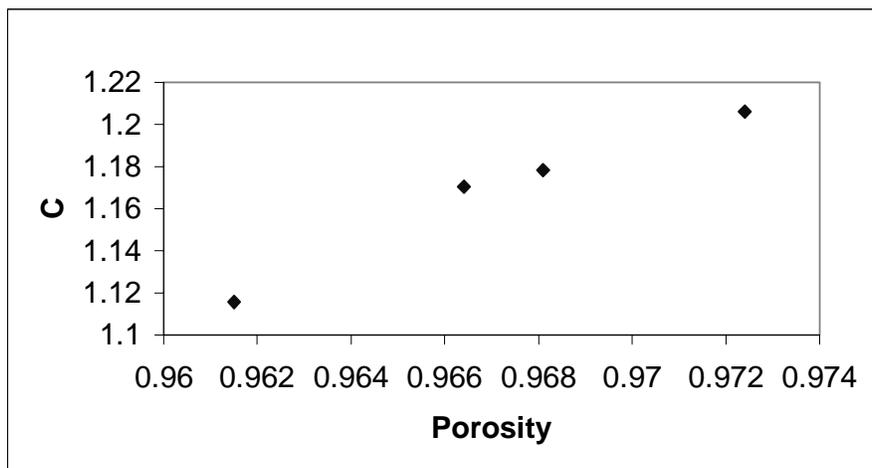


Figure 5. Variation of C with ϕ in RVC-air system

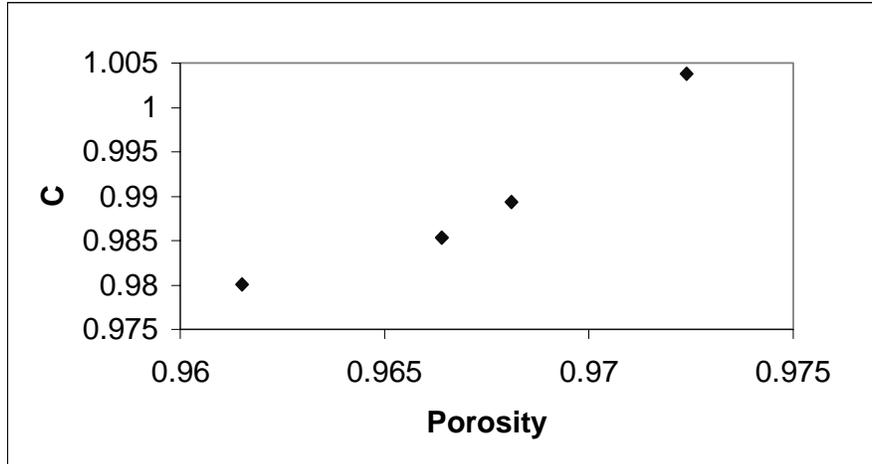


Figure 6. Variation of C with ϕ in RVC-water system

3. CALCULATIONS AND VALIDATION

To validate our empirical correlation, we considered four representative combinations of high porosity foam materials and fluid in the present calculations i.e. aluminium in air, aluminium in water, reticulated vitreous carbon (RVC) in air and reticulated vitreous carbon (RVC) in water [3]. The values of ETC has been calculated using equation (3) and compared the results as given in table 1. It has been observed that the calculated values of ETC are very close to the experimental results within average percentage deviation of just over 3.3. In table 2 experimental results of the ETC for the same samples are compared with the calculated values of the Boomsma's and Bhattacharya et al models. The expressions as used in the calculations have been given in the appendix. To confirm our proposition, the average percentage deviation from the experimental results for each model has been determined and mentioned at the bottom of the table 2. From both the tables it is found that the percentage deviation is least for our empirical proposition.

4. CONCLUSIONS

The empirical correlation given here is for highly porous foam like materials, but it is equally valid for the calculation of ETC for other types of materials. It has a single correlation factor, which is valid for all types of systems. It has potential for further refinements to predict ETC values of consolidated and reinforced composites.

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APPENDIX:

The various expressions and correction terms for Boomsma's model and Bhattacharya's are shown below:

The Boomsma's model (2001) is

$$\lambda_e = \frac{\sqrt{2}}{2 [R_A + R_B + R_C + R_D]} \quad (A)$$

Where $R_A = 4F / [\{2e^2 + \pi F (1 - e)\} \lambda_s + \{4 - 2e^2 - \pi F (1 - e)\} \lambda_f]$,

$R_B = (e - 2F)^2 / [(e - 2F) e^2 \lambda_s + \{2e - 4F - (e - 2F) e^2\} \lambda_f]$,

$R_C = (\sqrt{2} - 2e)^2 / [\{2\pi F^2 (1 - 2e\sqrt{2}) \lambda_s\} + 2\{\sqrt{2} - 2e - \pi F^2 (1 - 2e\sqrt{2})\} \lambda_f]$,

$R_D = 2e / [e^2 \lambda_s + (4 - e^2) \lambda_f]$,

$F = \sqrt{[\sqrt{2}\{2 - (5/8)e^3\sqrt{2} - 2(1 - \phi_1)\} / \{\pi(3 - 4e\sqrt{2} - e)\}]}$,

and $e = 0.339$.

Bhattacharya's model (2002) is

$$\lambda_e = F \{ \phi_f \lambda_f - (1 - \phi_f) \lambda_s \} + \frac{(1 - F)}{\{ \phi_f / \lambda_f - (1 - \phi_f) / \lambda_s \}} \quad (B)$$

Where $F = 0.35$.

Symbols involved in the formulae (A) and (B) have the same meaning as in the previous part in the paper.

Table 1. Comparison of ETC values for two-phase systems calculated using equation 3. The thermal conductivity is in $\text{W m}^{-1} \text{K}^{-1}$.

S. No.	Type of the sample	ϕ_f	λ_s	λ_f	λ_e (expt)	λ_e (theo)	% error
1	Al/air ¹	0.971	218.0	0.026	2.7	2.65	1.9
2	Al/air ¹	0.946	218.0	0.026	4.6	4.36	5.2
3	Al/air ¹	0.905	218.0	0.026	6.7	6.60	1.5
4	Al/air ¹	0.949	218.0	0.026	3.9	4.17	6.9
5	Al/air ¹	0.909	218.0	0.026	6.7	6.41	4.4
6	Al/air ¹	0.978	218.0	0.026	2.2	2.10	4.4
7	Al/air ¹	0.949	218.0	0.026	4.0	4.17	4.3
8	Al/air ¹	0.906	218.0	0.026	6.9	6.55	5.0
9	Al/air ¹	0.972	218.0	0.026	2.5	2.57	2.8
10	Al/air ¹	0.952	218.0	0.026	3.9	3.97	2.0
11	Al/air ¹	0.937	218.0	0.026	4.5	4.91	9.1
12	Al/water ¹	0.971	218.0	0.615	3.7	3.54	4.2
13	Al/water ¹	0.946	218.0	0.615	5.4	5.23	3.2
14	Al/water ¹	0.905	218.0	0.615	7.65	7.44	2.7
15	Al/water ¹	0.949	218.0	0.615	4.8	5.04	5.0
16	Al/water ¹	0.909	218.0	0.615	7.6	7.25	4.6
17	Al/water ¹	0.978	218.0	0.615	3.05	3.00	1.8
18	Al/water ¹	0.949	218.0	0.615	4.95	5.04	1.8
19	Al/water ¹	0.906	218.0	0.615	7.65	7.40	3.3
20	Al/water ¹	0.972	218.0	0.615	3.3	3.47	5.1
21	Al/water ¹	0.952	218.0	0.615	4.75	4.85	2.1
22	Al/water ¹	0.937	218.0	0.615	5.35	5.76	7.6
23	RVC/air ¹	0.9664	8.5	0.026	0.164	0.164	0.0
24	RVC/air ¹	0.9724	8.5	0.026	0.15	0.145	3.2
25	RVC/air ¹	0.9615	8.5	0.026	0.17	0.178	4.6
26	RVC/air ¹	0.9681	8.5	0.026	0.16	0.160	0.0
27	RVC/water ¹	0.9664	8.5	0.615	0.73	0.733	0.4
28	RVC/water ¹	0.9724	8.5	0.615	0.722	0.712	1.4
29	RVC/water ¹	0.9615	8.5	0.615	0.743	0.750	1.0
30	RVC/water ¹	0.9681	8.5	0.615	0.727	0.727	0.0
Average deviation							3.3 %

¹ Bhattacharya et al (2002).

Table 2. Comparison of ETC values for two-systems calculated using different models. Thermal conductivity is in $\text{W m}^{-1} \text{K}^{-1}$.

Sample Number	λ_e (expt.)	<u>Boomsma's model</u>		<u>Bhattacharya's model</u>	
		using eqn (A)	% error	using eqn (B)	% error
1	2.7	1.039	61.5	2.238	17.1
2	4.6	2.894	37.1	4.146	9.8
3	6.7	5.621	16.1	7.275	8.5
4	3.9	2.684	31.2	3.917	0.4
5	6.7	5.362	19.9	6.970	4.0
6	2.2	0.464	78.9	1.704	22.5
7	4.0	2.684	32.9	3.917	2.0
8	6.9	5.556	19.4	7.199	4.3
9	2.5	0.959	61.3	2.162	13.5
10	3.9	2.470	36.6	3.688	5.4
11	4.5	3.514	21.9	4.833	7.4
12	3.7	2.001	45.9	2.833	23.4
13	5.4	3.753	30.5	4.746	12.1
14	7.65	6.435	15.8	7.884	3.1
15	4.8	3.551	26.0	4.516	5.9
16	7.6	6.177	18.7	7.578	0.3
17	3.05	1.475	51.6	2.297	24.6
18	4.95	3.551	28.2	4.516	8.7
19	7.65	6.371	16.7	7.808	2.1
20	3.3	1.927	41.6	2.756	16.4
21	4.75	3.346	29.5	4.287	9.7
22	5.35	4.353	18.6	5.435	1.6
23	0.164	0.094	42.4	0.126	23.0
24	0.15	0.077	48.4	0.108	27.7
25	0.17	0.108	36.4	0.141	17.1
26	0.16	0.089	43.9	0.121	24.3
27	0.73	0.772	5.8	0.720	1.3
28	0.722	0.753	4.3	0.701	2.8
29	0.743	0.788	6.1	0.736	0.9
30	0.727	0.767	5.5	0.715	1.6
Average deviation			31.1%		10.1%